Number Games:

Pandigital Numbers, Friedman Numbers, and $e$

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Slides and worksheets can be found at:
http://www.ma.utexas.edu/users/richard.wong/Notes.html
Pandigital Numbers

Definition

A pandigital number is an integer that uses each digit 0-9 exactly once in the significant digits of its decimal representation.

Example

- 1234567890 is a pandigital number.
- 0123456789 is not.
- 11234567890 is not a pandigital number, but it is a pandigital number with redundant digits.
Pandigital Numbers

Definition

A pandigital number with redundant digits is an integer that uses each digit 0-9 at least once in the significant digits of its decimal representation.

Definition

A pandigital number is an integer that uses each digit 0-9 exactly once in the significant digits of its decimal representation.
Work on the Pandigital numbers section of the worksheet!
Friedman numbers

**Definition**

A **Friedman number** is an integer that can be non-trivially expressed as a formula using each of its significant digits exactly once, along with the operations (+, −, ×, ÷), additive inverses, parentheses, and exponentiation.

**Example**

- \((n)\) is a trivial way to express an integer \(n\). So that means that the single digit numbers cannot be Friedman numbers.
- 343 is a Friedman number, since \((3 + 4)^3 = 343\).
Work on the Friedman numbers section of the worksheet!
Euler’s constant

So far today we have been investigating numbers and their decimal representations.

However, we will now investigate a number that is **irrational**, and even **transcendental**.

\[ e = 2.718281828459045 \ldots \]
However, we don’t need to know the significant digits of a number to define it.

\[ \sum_{k=0}^{\infty} \frac{1}{k!} = e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \]

We can use these formulas (among others) to approximate the digits of \( e \).
Work on the Approximating $e$ section of the worksheet!
How closely do you think we can approximate \( e \) using a pandigital formula, using the operations \( (+, -, \times, \div) \), additive inverses, parentheses, and exponentiation?
A pandigital formula

\[ e \approx (1 + 9^{-4 \times 7})^{3^{285}} + 0 \]

This formula was found by Richard Sabey in 2004, and is correct to \(18 \times 10^{24}\) digits.

That’s 18 trillion \(trillion\) digits!
The inspiration

\[ e \approx (1 + 9^{46 \times 7})^{3^{285}} + 0 \]

- First recall that \( e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \).
- Now note that \( 3^{2^{85}} = 9^{2^{84}} = 9^{4^{42}} 9^{4^{6 \times 7}} \).
- Then, set \( n = 3^{2^{85}} \approx 1.846 \times 10^{25} \).
The error term

To approximately determine the error term, we need some analysis.

Note that \((1 + \frac{1}{n})^n < e < (1 + \frac{1}{n})^{n+1}\).

Therefore,

\[
e - \left(1 + \frac{1}{n}\right)^n < \left(1 + \frac{1}{n}\right)^{n+1} - \left(1 + \frac{1}{n}\right)^n = \left(1 + \frac{1}{n}\right)^n \left(\frac{1}{n}\right) = \frac{e}{n}
\]

Hence, given a choice of \(n\), the error term is less than \(\frac{e}{n}\).