What's it like being a mathematician? How can you become one?

Richard Wong

Connected Research at UCLA

Slides available at math.ucla.edu/~richardwong

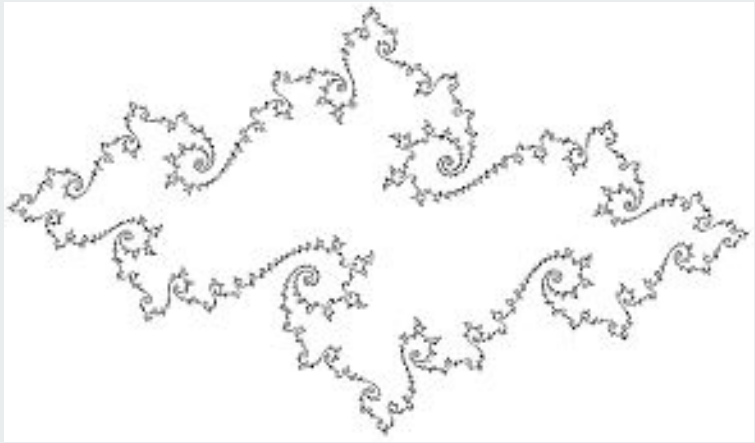
What is math research like?

"It seems to me that the poet has only to perceive that which others do not perceive, to look deeper than others look. And the mathematician must do the same thing."
- Sofya Kovalevskaya

"In mathematics, the art of asking questions is more valuable than solving problems." - Georg Cantor

Does every planar, continuous simple closed curve contain all four vertices of some square?

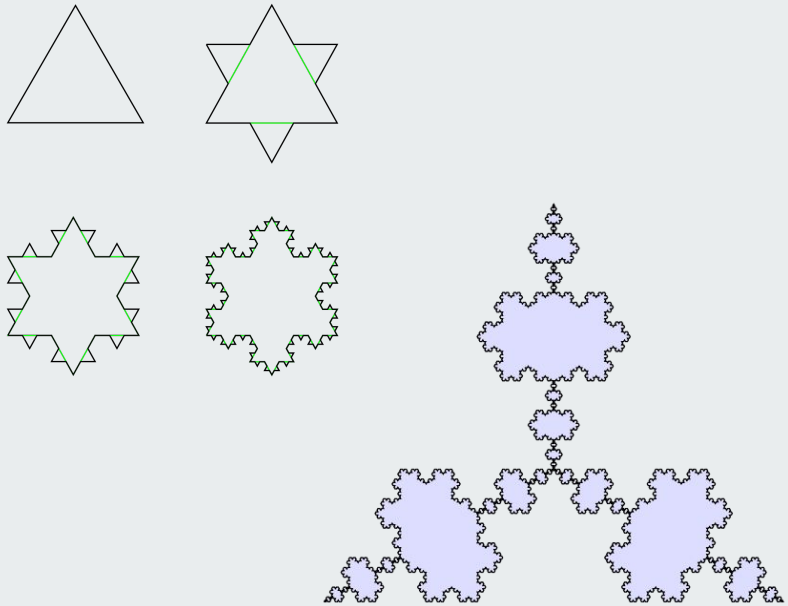
This is known as the Square Peg Problem.



Does every plane simple closed curve contain all four vertices of some square?

The Koch Snowflake is obtained from a equilateral triangle by recursively altering each line segment by

1. divide the line segment into three segments of equal length.
2. draw an equilateral triangle that has the middle segment from step 1 as its base and points outward.
3. remove the line segment that is the base of the triangle from step 2.



“Everyone knows what a curve is, until he has studied enough mathematics to become confused through the countless number of possible exceptions.”
- Felix Klein

- **1911** - Problem proposed by Otto Toeplitz
- **1916** - Proved for piecewise analytic curves (Emich)
- **1921, 1950 -** Convex curves (Zindler, Christensen)
- **1961** - Analytic curves (Jerrard)
- **1989** - Locally monotone curves (Stromquist)
- **1995** - Curves symmetric across a line or about a point (Nielsen-Wright)
- **2017** - The union of graphs of two Lipschitz functions (Tao)

How can we generalize this problem?

- For a given Jordan curve, how many inscribed squares are there?
- Does every Jordan curve contain all three vertices of some triangle? Of some equilateral triangle?
- Does every Jordan curve contain all four vertices of some rectangle?
- For a given quadrilateral, does every Jordan curve contain all four vertices of a similar quadrilateral?
- Does every Jordan curve contain all n vertices of some n-gon (for $n > 5$)?

Question break!

Things to consider about math graduate programs:

Challenges:

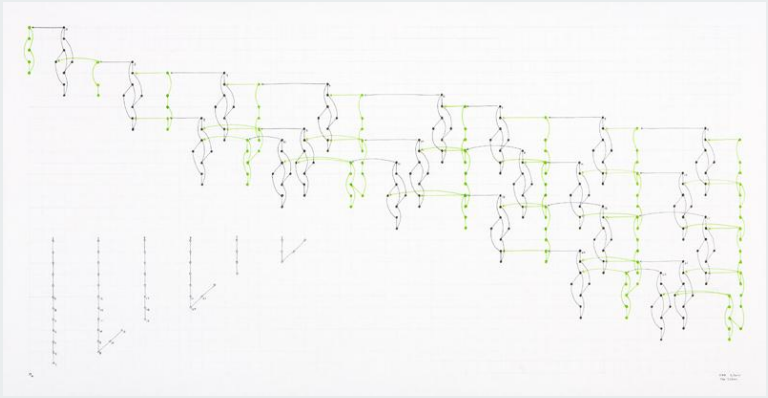
- Opportunity cost
  - What are your career goals?
- Job market
- Moving to a new location
- Finding a support network
- Finding an advisor
- Passing prelim exams

Benefits:

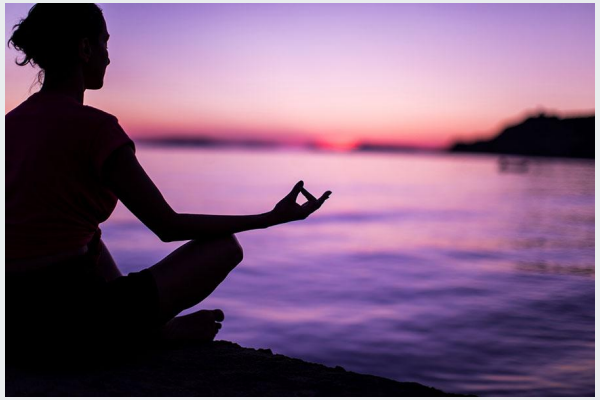
- Get paid to learn/research
  - Academic/intellectual freedom
  - Research seminars
  - Teaching / professional development
- Mathematical community
  - Outreach
- Conference travel

Advice for pursuing a Ph.D. in mathematics:

- Immerse yourself in mathematics



- Sustain yourself and avoid burnout



Advice for pursuing a Ph.D. in mathematics:

❖ Immerse yourself in mathematics

- ❖ Ask questions!
- ❖ Go to seminar talks.
  - ➢ Ravi Vakil's three things exercise
- ❖ Go to conferences.
  - ➢ Talk to experts outside of your university.
  - ➢ Advertise yourself and your projects

❖ Sustain yourself and avoid burnout

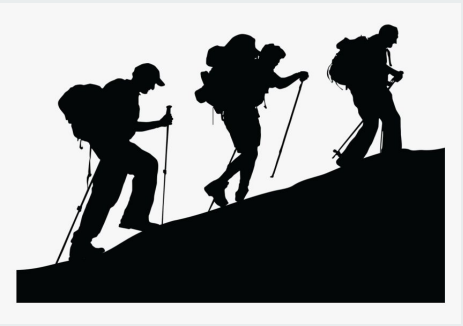
- ❖ Find a support network:
  - ➢ Find multiple mentors
  - ➢ Find peers to work with
- ❖ Make time for non-math hobbies!
- ❖ Getting a Ph.D. is a marathon, not a race.
- ❖ The best Ph.D. is a finished Ph.D.

Skills to develop as a mathematician:

Questioning Skills

Collaboration Skills

Perseverance



Communication Skills

Question break!

Advice on applying to math graduate programs:

These are recommendations, not requirements!

- Take **graduate courses** in mathematics.
- Participate in research-like opportunities
  - Directed Reading Program
  - Research Experience for Undergraduates (REUs)
  - Reading courses
- Develop strong relationships with tenured/tenured-track faculty.
  - **Letters of recommendation** are important!
- Get involved in extracurricular opportunities!

Questions for you to ask about a math graduate program:

- Can you succeed at this institution?
- What support will you have?

Questions for you to ask about a math graduate program:

- Can you succeed at this institution?
  - What potential advisors are there?
    - Are they taking students?
    - Have their students been successful?
  - What is the department culture like?
    - Are there seminars in your area?
    - Are the graduate students happy?
- What support will you have?
  - Fellowship/TA support?
  - Conference/Travel support?
  - How many years of funding?
  - What professional development support is there?

Thanks for listening!

References:

New Geometric Perspective Cracks Old Problem About Rectangles, Kevin Hartnett

A Survey on the Square Peg Problem, Benjamin Matschke

An integration approach to the Toeplitz square peg problem, Terence Tao