Math 32AH: Calculus of Several Variables
Fall 2022, Lecture 1

Instructor:	Richard Wong (He/Him)	**Lectures:**	MWF 9-9:50am
Email:	richardwong@math.ucla.edu	**Place:**	Royce 150
Office:	MS 6304	**Canvas:**	Website here.

Content Office Hours: TBD; or by appointment
Social Office Hours: TBD

"It seems to me that the poet has only to perceive that which others do not perceive, to look deeper than others look. And the mathematician must do the same thing." — Sofya Kovalevskaya

Contents

The dates and topics in this syllabus are subject to change. Any changes will be announced on Canvas.

Course Description

How can we describe the physical world mathematically? What changes, and what stays the same when we move from single variable calculus to multivariable calculus? What does it mean to take a derivative of a multivariable function?

Multivariable calculus is the mathematical language that allows us to describe the geometry of the physical world around us, such as the motion of planets in orbit, the behavior of electromagnetic forces, or the path of steepest ascent through the hills of Los Angeles. In this course, you will develop the reasoning and questioning skills needed to explore these geometric concepts and apply them to real-life situations. Moreover, you will become fluent in communicating your ideas through the mathematical language of multivariable calculus.

The course 32AH differs from 32A in that it covers the topics of multivariable calculus with more mathematical rigor. Moreover, it builds the foundation for more advanced topics, such as real analysis, complex analysis, and differential geometry. This course is recommended for students interested in learning about advanced mathematics.

Prerequisites

Math 31A (Differential and Integral Calculus) with a grade of B or better.

Course Materials

- *Calculus and Analysis in Euclidean Space*, by Shurman.
- *Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach*, by Hubbard and Hubbard.
- *Honors Multivariable Calculus* Lecture Notes.

The official course textbook is by Shurman. I also recommend Hubbard and Hubbard - you might find it convenient to obtain copies of these books for reference. **However, these books are not required**. Instead, I will be providing lecture notes for the course, which can be found here. Homework will be assigned directly from the course lecture notes.

- Recorded class lectures and lecture slides, will be made privately available on Canvas. Please note that these are intended to supplement lecture, and that **these materials not a suitable replacement for attending live lectures.**
- We will also use Campuswire as a tool for asking and answering questions, as well as working collaboratively.

Learning Objectives

"In mathematics, the art of asking questions is more valuable than solving problems." — Georg Cantor

The goals of the course are that you:

(1) acquire an understanding of the geometry of Euclidean space, linear transformations, and the differential calculus of multivariable functions;

(2) develop the questioning, reasoning, and proof skills needed to explore mathematical ideas and topics, and to apply them to real-life situations;

(3) develop the collaboration and communication skills needed to convey your (mathematical) ideas.

Furthermore, this course is designed to show you that mathematics is neither a "spectator sport", nor a solitary endeavor. Mathematics is both a creative and a collaborative process, and *everyone*, especially you, can do mathematics and be a part of the mathematical community. My hope is that by the end of the semester, you will be proud of the mathematics that you have done in this course.

"I don't solve quadratic equations to help me with my daily life, but I do use mathematical thinking to help me understand arguments and to empathize with other people. And so pure maths helps me with the entire human world." — Eugenia Cheng

Lecture Schedule

	Learning Outcome:	**Lectures:**
MV1:	$\mathbb{R}^n$ **as a vector space.** Geometrically interpret and reason with vector space axioms. Prove that collections are vector spaces or subspaces. Determine if a set of vectors are linearly independent, or a basis. Calculate coordinate vectors.	1-3
MV2:	**Linear maps and matrices.** Determine if a function from $\mathbb{R}^n \to \mathbb{R}^m$ is linear. Determine the standard matrix associated to a linear map. Add and multiply matrices. Verify that a linear map is invertible. Compute determinants of 2×2 and 3×3 matrices.	3-5
MV3:	**Vector operations in** $\mathbb{R}^n$**.** Compute and interpret dot products, the projection of a one vector onto another vector, and the angle between two vectors. State and use the Cauchy-Schwarz and triangle inequalities. Determine the equations of lines and planes in space. Compute cross products. Compute the volume of a parallelpiped.	6-9
	Midterm 1	**Oct. 17**
MV4:	**Analysis in** $\mathbb{R}^n$**.** Sketch and geometrically interpret vector-valued and multivariable functions. Compute limits of sequences, limits of multivariable functions; determine if multivariable functions are continuous.	10-15
MV5:	**The multivariable derivative.** Geometrically interpret the derivative of a multivariable function. Compute and use partial derivatives and the Jacobian matrix. Find equations for tangent planes to surfaces and linear approximations of functions at a given point. State and use the chain rule for derivatives.	16-19
	Midterm 2	**Nov. 14**
MV6:	**Local optimization.** Compute directional derivatives and the gradient. Interpret level curves of multivariable functions. Find and classify local extrema of a multivariable function.	20-22
MV7:	**Constrained optimization.** Determine if subsets of $\mathbb{R}^n$ are open, closed, bounded, or compact. Use the method of Lagrange multipliers to find local minima and local maxima of functions subject to constraints. Find and classify global extrema on compact domains. Compute and use the Hessian matrix.	23-25
	Final Exam	**Dec. 6**

Course Structure

"The only way to learn mathematics is to do mathematics." — Paul Halmos

This course is offered in an in-person, synchronous format. Lectures, discussion sections, and office hours will be held in person. All exams will be held in person, during class hours.

During class lectures, I plan to use a mix of direct teaching (aka traditional lecturing), as well as active and inquiry-based teaching. Tasks you will be asked to do include: work individually, work in small groups, discuss ideas in small groups, ask questions, and/or present your ideas or solutions to the class. The format of the discussion sections are largely left to the individual TAs.

The Learning Process

Each assignment in this course plays an important role in the learning process. See below for a detailed description of each assignment!

1. **Pre-class work:** Before coming to lecture, you should download the lecture slides, and read the relevant chapter or section of the textbook/lecture notes. You should not expect to understand everything immediately - this is completely normal! You should bring any questions you have to lecture (and office hours).

2. **Lecture:** During lecture, I will explain and motivate the material by providing examples, geometric intuition, and the context of the material. I will also provide problems for you to actively practice on in small groups.

3. **Post-class work:** After lecture, you should review your notes and/or the lecture slides, and complete as many homework problems as you need in order to become comfortable with the material. If you have questions at this stage, you should ask them in office hours, or on Campuswire!

4. **Quizzes:** In discussion section, the individual quiz is your first opportunity to simulate a test-like environment and assess your understanding of the material. The group quiz is designed to help you learn and master the material, as well as develop your mathematical communication skills. Being able to explain concepts or solutions to your peers is a great way to assess your understanding of the material.

5. **Challenge Problem Reports / Exams:** These assignments are for you to demonstrate your mastery of the material, and will emphasize critical thinking, rather than memorization of the material. That is, these assignments will emphasize applying what you've learned to new and unique situations. By this point, you should feel comfortable enough with the material to answer complex questions and/or explain concepts in depth.

6. **Reflection:** After any major assignment, it's important to (1) review the feedback on your work, (2) think about what went well, and (3) what changes you need to make (e.g. in your study strategies, your understanding of the material, etc.). This will help you improve on future assignments!

Major Assignments

1. **Homework:** Each lecture will have accompanying homework problems assigned from the lecture notes. These problems are chosen to help you best understand the material. You are encouraged to work together on the homework problems.

   Assigned homework problems will be due roughly every week, and **will be graded on a credit/no credit system**. You will receive credit for the assignment as long as you make a reasonable attempt at the assigned problems.

2. **Quizzes:** There will be a total of 7 two-part cumulative quizzes given during the discussion sections. In the first part, you will attempt the quiz individually, without any outside resources. In the second part, you will work on the quiz in small groups, with any resources at your disposal. You will submit both parts of the quiz to be graded, but **only the group quiz will count towards your course grade**. The group solution should not be a duplicate of any individual submission.

   The problems will cover topics from any past lecture or homework assignment. The individual part is designed to help you assess your current understanding of the material. The group part is designed to help you learn and master the material, as well as develop your mathematical communication skills.

3. **Challenge Problem Reports:** There will be a total of 4 "Challenge Problem Reports" given during class and discussion. These are technical writing assignments, where you will explore concepts in multivariable calculus. You will start working on these in small groups, and you will have roughly a week to individually submit a polished write up of your solutions. These assignments are designed to develop your mathematical reasoning and communication skills.

Exams

There will be two non-cumulative 50 minute midterms, and one cumulative 3-hour final. All exams will be held in person, during class hours. The topics covered on the midterms depends on the schedule, and will be announced one week in advance of the midterm.

You must bring a photo ID to the exams. Calculators will not be allowed on exams. For each exam, you will be allowed to bring a handwritten reference sheet. For the midterms, the reference sheet must be be written on a single 3" x 5" index card (double-sided). For the final, the reference sheet may consist of three 3" x 5" index cards (double-sided). Sheets that do not meet these requirements will be confiscated at the beginning of the exam.

Midterm 1	Monday Oct 17	9-9:50am	Royce 150
Midterm 2	Monday Nov 14	9-9:50am	Royce 150
Final	Tuesday Dec 6	11:30am-2:30pm	TBD

It is okay to ask for an alternate examination time, e.g. if you have a course conflict. I will do my best to accommodate alternate examination times. If you have a conflict, please let me know ASAP, **up to two weeks in advance of the exam,** so I can take care of the logistics.

If you are sick, or need to quarantine during an exam period, you should not come to the exam. You should let me know ASAP, and we will proceed on a case-by-case basis. If you miss

a midterm, then your grade will automatically be computed by grading scheme 2 as outlined below (Grades).

You must take the final exam in order to pass the course. The UCLA student handbook outlines when a make-up final exam is permitted.

Reflection Assignments

As noted in the course learning objectives, reflection is an important step in the learning process. As such, there are multiple **optional** assignments in this course for you to reflect on your learning strategies and look for ways to improve.

1. **Pre-Midterm 1 Survey:** This survey assesses your preparation and study strategies prior to Midterm 1. If you complete this optional assignment, you will earn 0.25% extra credit on your final grade.
2. **Post-Midterm 1 Survey:** This survey allows you to reflect on Midterm 1, and helps you reflect on how you will prepare for future exams. If you complete this optional assignment, you will earn 0.25% extra credit on your final grade.
3. **Challenge Problem Report 1 Revision and Reflection:** After Challenge Problem Report 1 is graded, you will have the opportunity to reflect on your work and revise your solutions. If you complete this optional assignment, your grade on the revised solutions **replaces** your original Challenge Problem Report 1 grade. Your letter grade will never be lowered by this assignment. However, it will not necessarily be improved.
4. **End of Quarter Reflection:** This is an **optional** writing assignment to be submitted before the final exam. It is designed for you to reflect on what you have learned and achieved in this course.

   If you complete this assignment and your course grade is close to a higher grade bracket, I will additionally take into account how thorough, detailed, and thoughtful your responses are to this assignment when assigning a letter grade. Otherwise, your final grade will be determined as outlined below. Your grade will never be lowered by this assignment.

Grades

"You need to have a conversation with yourself about what is important for you, what you actually need to thrive. And to not fall prey to the belief system that the only thing of value is your mathematics." — Pamela Harris

Your grade will reflect your performance in the course using the **better** of the following two grading schemes. This will happen automatically.

Scheme 1		**Scheme 2**	
Homework	9%	Homework	9%
Quizzes	21%	Quizzes	21%
Challenge Problems	24%	Challenge Problems	32%
Midterm 1	12%	Best Midterm grade	12%
Midterm 2	12%		
Final	20%	Final	26%

A letter grade will be assigned to percentages via the following brackets.

A	A-	B+	B	B-	C+
[100, 93]	(93, 90]	(90, 87]	(87, 83]	(83, 80]	(80, 77]

C	C-	D+	D	D-	F
(77, 73]	(73, 70]	(70, 67]	(67, 63]	(63, 60]	(60, 0]

Borderline grades may be improved by submitting the **End of Semester Reflection** as outlined above. I reserve the right to award an A+ for exceptional performance. I also reserve the right to adjust the grade cutoffs dependent on overall class scores at the end of the semester. This will only ever make it *easier* to obtain a certain letter grade.

Policies

Student Conduct / Classroom Environment Policy

Everyone can have joyful, meaningful, and empowering mathematical experiences. — Federico Ardila

I strongly believe that EVERYONE is capable of success in this course and in mathematics in general, regardless of the systemic barriers that exist due to race, gender, socio-economic background, or cultural identity.

In fact, I hope to show you all that mathematics can be inspiring, affirming, and even empowering. I strive to create positive and inclusive learning environments where all students feel welcome to ask questions and to voice their ideas. In particular,

- **You belong in this classroom.** Discrimination or harassment of any kind will not be tolerated. Please let the instructor know immediately if you ever feel uncomfortable in class. You may report an incident to the Office of Equity, Diversity and Inclusion here.
- **You deserve to be addressed in the manner that reflects who you are.** If you are comfortable with it, you are welcome to share your pronouns and/or preferred name at any time. Conversely, please address your classmates according to their expressed preferences.
- **You deserve to fully and equitably participate in our learning environment.** During class, I encourage you to interrupt me with questions at any time! Please let me know as soon as possible if you need any classroom accommodations.
- **Be comfortable with asking questions and making mistakes.** Doing so is an essential part of the learning process, and no question is too basic or stupid. I ask you all to respect and be patient with your peers when they are confused.

While studying mathematics can often be challenging intellectually, it can be challenging *emotionally* as well. In my experience, having a strong support network of teachers, mentors, colleagues, peers, and friends that can support you is the best way to help you persevere and succeed in mathematics. To help build an empathetic support network in class, I ask that you all:

- Reach out to people you do not know and actively build new connections;

- Respect and understand that different people may bring differences in background, expertise, and interest;
- Assume the best in others and give them the benefit of the doubt. However, understand that behavior can have an adverse impact on others, even in the absence of malicious intent.
- Do not interrupt your peers; demean them or their ideas; or challenge their competence or mathematical abilities.

Covid-19 Policy

Ensuring a safer campus depends on each of us following the latest UCLA health and safety guidelines. While campus policies must be modified to address changing local, state, and national orders and guidance, the most current information is available at covid-19.ucla.edu.

Calculator Policy

You are welcome to use calculators or Wolfram Alpha (a free online calculator) on homework, group quizzes, and challenge problems. However, no calculators will be allowed on any of the exams (or individual quizzes) in this course. You are expected to be able to perform basic arithmetic operations with fractions and decimals by hand, and know common values of trigonometric and log functions.

Attendance Policy

Attendance at our MWF lectures is strongly recommended, but not required. All required information will be made available through lectures (which will be recorded and posted on Canvas). However, consistently attending lecture is the best way to ensure that you do not fall behind in class.

There is a Zoom link for lectures, which will allow for you to join the meeting virtually if you are unable to attend lecture, but note that this is not a suitable replacement for attending live lectures. In particular, I will not be paying attention to the chat, nor will I be able to hear any virtual questions.

Attendance at your discussion section is also strongly recommended, and it is required in the sense that assignments that affect your grade will be given during discussion. If you happen to miss a discussion section for whatever reason, see the late/missed assignment policy.

Late/Missed Assignment Policy

Sometimes we have bad days, bad weeks, or bad quarters. This is especially true in light of the COVID-19 pandemic, and this crisis, as well as any other unexpected, unfortunate personal crisis, should not unduly affect your grade.

- **Homework:** There is a 48-hour grace period for submitting homework after the posted deadline. You do not need to notify me or your TA of your use of this grace period. Since homework is graded only on completion, no late homework will be accepted after this 48-hour late deadline.
- **Quizzes:** If you happen to miss a quiz once or twice, you should first contact your TA to let them know. You can make up a missed group quiz on your own by completing and uploading the quiz via Gradescope. Note that you can use any of your notes and resource

to complete the group quiz. There is no penalty as long as the group quiz is submitted **before the next discussion section**.

- For **challenge problems,** I am using the policy of "time banks" to accommodate any unexpected issues. You may use this policy one of two ways (please choose, and let me know):
  - You may have one 48-hour grace period for one submitted assignment, OR
  - You may have two 24-hour extensions for two different submitted assignments.

  You do not have to justify your use of this policy, nor do you need to use it at all. You will not be penalized for using the time bank policy.

However, if you are having consistent problems keeping to the schedule, or if you find yourself struggling with unexpected personal events, I encourage you to reach out and email me (richardwong@math.ucla.edu) as soon as possible. I can also give case-by-case flexibility depending on the severity of the issues.

Regrading Policy

Occasionally, I, your TAs, or the graders may make a mistake while grading assignments or exams. If there has been a clerical error (e.g. there was an error in calculating the points you earned, or an error in recording the grades on Canvas), you can contact me or the TAs immediately to fix this error.

For all other grading issues, you should submit a Request for Regrade Form to Canvas anytime within the regrading window for the assignment. Unless announced otherwise, this window lasts for one week, beginning 24 hours after the assignment or exam has been returned.

Please note that the regrading policy is intended to fix serious errors in grading, *not* to argue for extra points. Your grade will not necessarily be improved by the regrade.

Academic Integrity

UCLA is a community of scholars. In this community, all members including faculty, staff and students alike are responsible for maintaining standards of academic honesty. As a student and member of the University community, you are here to get an education and are, therefore, expected to demonstrate integrity in your academic endeavors. You are evaluated on your own merits. Cheating, plagiarism, collaborative work, multiple submissions without the permission of the professor, or other kinds of academic dishonesty are considered unacceptable behavior and will result in formal disciplinary proceedings usually resulting in failing the course, suspension or dismissal. See the Dean of Students website for more information.

Office Hour Policy

Office hours will be held both in person and virtually. In addition to the traditional "course content" office hours, I will also hold "social" office hours for you to get to know me, your classmates, your peers at UCLA, and (occasionally) professional mathematicians.

You are strongly encouraged to come to office hours, both with me and your TA. You might find the course content office hours most helpful if you have specific questions prepared, but I also welcome you to come and listen to your peers' questions. For the social office hours, there will often be a topic or reading to guide the conversation.

Contact / Email policy

If you have a course-related question, you are strongly encouraged to post in the course Canvas or CampusWire before emailing me. Others might be able to answer your question, and others might find the answer to your question helpful as well.

The best way to contact me is via email (richardwong@math.ucla.edu). To ensure that I see your email, the subject line should include the phrase "**Math 32AH**". To ensure I know who you are, the message or signature should include **your name** and **UCLA University ID**.

I will do my best to respond to your email in a timely manner (typically within a few hours). However, if you send an email during the evening or the weekend, do not expect to hear a response until the next weekday morning.

Resources

Services for Students with Disabilities

I am committed to creating an accessible and inclusive learning environment. Please let me know if you experience any barriers to learning so that I can work with you to ensure you have equal opportunity to participate fully in this course.

If you are already registered with the Center for Accessible Education (CAE), please request your Letter of Accommodation in the Student Portal. If you are seeking registration with the CAE, please submit your request for accommodations via the CAE website. Students with disabilities requiring academic accommodations should submit their request for accommodations as soon as possible, as it may take up to two weeks to review the request. For more information, please visit the CAE website, visit the CAE at Murphy Hall A255, or contact them by phone at (310) 825-1501.

Further Resources

- UCLA has a multitude of groups, resources, and services available to support your academic success, your social belonging, your physical and mental health, and your overall well-being. You can explore those resources here.
- UCLA provides resources if you are feeling overwhelmed and need personal and/or academic assistance. You can find a list of resources organized by need here.
- If you are experiencing a financial crisis that impacts your academic success at UCLA, the Economic Crisis Response Team (ECRT) may be a helpful resource. ECRT aims to efficiently, compassionately and discreetly offers a seamless and individualized response to UCLA students in extraordinary financial crisis. You may submit a self-assessment form by visiting this link. The self-assessment form will allow the team to assess options and provide resources best suited to address your needs.

Title IX

Please note that Title IX prohibits gender discrimination, including sexual harassment, domestic and dating violence, sexual assault, and stalking. If you have experienced sexual harassment or sexual violence, you can receive confidential support and advocacy at the CARE Advocacy Office for Sexual and Gender-Based Violence, 205 Covel Commons, Los Angeles, CA, 90095; care@careprogram.ucla.edu; (310) 206-2465.

Counseling and Psychological Services (CAPS) provides confidential counseling to all students and can be reached 24/7 at (310) 825-0768.

I am required under the UC Policy on Sexual Violence and Sexual Harassment to inform the Title IX Coordinator should I become aware that you or any other student has experienced sexual violence or sexual harassment. You can also report sexual violence or sexual harassment directly to the University's Title IX Coordinator, 2255 Murphy Hall, titleix@equity.ucla.edu, (310) 206-3417. Reports to law enforcement can be made to UCPD at (310) 825-1491.